ĐÁP ÁN

Câu hỏi	Nội dung	**Điểm**
Câu 1		***1,5đ***
	$(S): x^2+y^2+z-10=0$ $\vec{n}=(\frac{\partial F}{\partial x},\frac{\partial F}{\partial y},\frac{\partial F}{\partial z})=(2x,2y,1)$ Tại $M(2;-2;2)$: $\vec{n}=(4;-4;1)$	***0,5đ***
	Phương trình tiếp tuyến (Δ): $\frac{x-2}{4}=\frac{y+2}{-4}=\frac{z-2}{1}$	***0,5đ***
	Phương trình pháp diện $(\alpha): 4(x-2)-4(y+2)+1(z-2)=0$ hay $4x-4y+z-18=0$	***0,5đ***
Câu 2		***2đ***
	a) $div\vec{F}=-36e^{6x+8y}\cos(10z)-64e^{6x+8y}\cos(10z)+100e^{6x+8y}\cos(10z)$ $=0$	***0,5đ***
	b) $rot\vec{F}=(80e^{6x+8y}\sin(10z)-80e^{6x+8y}\sin(10z))\vec{i}$ $+(60e^{6x+8y}\sin(10z)-60e^{6x+8y}\sin(10z))\vec{j}$ $+(-48e^{6x+8y}\cos(10z)+48e^{6x+8y}\cos(10z))\vec{k}=\vec{0}$	***0,5đ***
	Suy ra $\vec{F}(x,y,z)$ là trường thế.	***0,5đ***
	c) Vì $div\vec{F}=0$ và $rot\vec{F}=\vec{0}$ nên $\vec{F}(x,y,z)$ là trường điều hòa.	***0,5đ***
Câu 3		***2đ***
	$S=\iint_S ds=\iint_D\sqrt{1+4x^2+4y^2}\,dxdy$, với $D: 1\le x^2+y^2\le 4$	***1đ***
	Đặt $\begin{cases}x=r\cos\varphi\\ y=r\sin\varphi\end{cases}$, $\lvert J\rvert=r \Rightarrow D:\begin{cases}0\le\varphi\le 2\pi\\ 1\le r\le 2\end{cases}$ $S=\int_0^{2\pi}d\varphi\int_1^2\sqrt{1+4r^2}.rdr$	***0,5đ***
	$=\frac{\pi}{6}(17\sqrt{17}-5\sqrt{5})$	***0,5đ***

<table>
<tr><td colspan="2">Câu 4</td><td>3,5đ</td></tr>
<tr><td></td><td>

a) Hoàn lưu H = $\int_C \left(xy^2 - 4y - y^2 \right)dx + (3x + x^2 y)dy$

Áp dụng công thức Green

$$H = \int_C \underbrace{(xy^2 - 4y - y^2)}_{P}dx + \underbrace{(3x + x^2 y)}_{Q}dy = \iint_D (7 + 2y)dxdy$$

$$= 7\iint_D dxdy + \iint_D 2ydxdy$$

$$= 7\,S(D) \;+ 0 = 7\,\pi 2^2 = 28\pi$$

($\iint_D 2ydxdy = 0$ vì $2y$ là hàm lẻ theo biến y và D đối xứng qua đường thẳng y = 0)

</td><td>

0,25đ

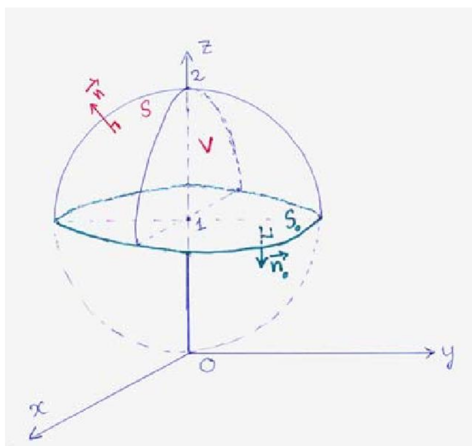
0,5đ

0,75đ

</td></tr>
<tr><td></td><td>

b)Thông lượng

$$W = \iint_S (x + ye^z)dydz + (y + xe^z)dxdz + (1 + 2z)dxdy$$



$$W = \iint_S + \iint_{S_o} - \iint_{S_o} = \oiint_{S \cup S_o} - \iint_{S_o}$$

Áp dụng công thức Gauss – Ostrogratski

$$\oiint_{S \cup S_o} = \iint_{S \cup S_o} \underbrace{(x + ye^z)}_{P}dydz + \underbrace{(y + xe^z)}_{Q}dxdz + \underbrace{(1 + 2z)}_{R}dxdy$$

$$= \iiint_V \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right)dxdydz = \iiint_V (1 + 1 + 2)dxdydz = 4V = 4\frac{2}{3}\pi 1^3 = \frac{8\pi}{3}$$

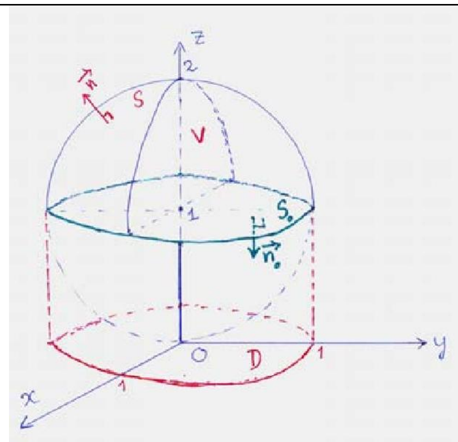
</td><td>

0,5đ

0,5đ

0,5đ

</td></tr>
</table>

<table>
<tr>
<td></td>
<td>


$S_o : z = 1 \Rightarrow dz = 0$

$\oiint_{S_o} = \iint_{S_o} \underbrace{(x + ye^z)}_{P} dydz + \underbrace{(y + xe^z)}_{Q} dxdz + \underbrace{(1 + 2z)}_{R} dxdy = \iint_{S_o} \underbrace{(1 + 2z)}_{R} dxdy$

$= -\iint_D (1 + 2.1) dxdy = -3S(D) = -3\pi 1^2 = -3\pi$

Suy ra: $W = \frac{8\pi}{3} - (-3\pi) = \frac{17\pi}{3}$
</td>
<td>0,5đ</td>
</tr>
<tr>
<td colspan="2">Câu 5</td>
<td>1đ</td>
</tr>
<tr>
<td></td>
<td>
$a_o = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)dx = \frac{1}{\pi} \int_{-\pi}^{\pi} (1 - |x|)dx = \frac{2}{\pi} \int_{0}^{\pi} (1 - x)dx = 2 - \pi$

$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nxdx = \frac{1}{\pi} \int_{-\pi}^{\pi} (1 - |x|) \cos nxdx$

$= \frac{2}{\pi} \int_{0}^{\pi} (1 - x) \cos nxdx = \frac{2}{\pi . n^2} (1 - \cos(n\pi))$, n = 1, 2, 3, ...

$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nxdx \ \frac{1}{\pi} \int_{-\pi}^{\pi} (1 - |x|) \sin nxdx = 0$, n = 1, 2, 3, ...

f(x) ~ $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx = \frac{2 - \pi}{2} + \sum_{n=1}^{\infty} \frac{2(1 - \cos(n\pi))}{\pi . n^2} \cos nx$
</td>
<td>0,5đ

0,5đ</td>
</tr>
</table>

Hết